portions of the rift. The King's Bowl explosion pit (Fig. 15-8) was the center of greatest violence. Lava drained to unknown depths below the water table, leaving the rift momentarily empty. Flooding ground water then resulted in steam explosions. A well recently drilled near the King's Bowl has encountered the present water table at a depth of 775 feet.

Fine ejecta was swept eastward by a west wind, obscuring the east edge of the lava field immediately down wind from the King's Bowl (Fig. 15-4). Nearby are small mounds which are believed to be rootless vents and erupted lava which flowed under the surface crust of flows from the King's Bowl vent. This surface originally was level with or higher than the tops of these rootless vents. Evidently much lava flowed out from these vents, enlarging the field, but great quantities must have drained back into the rift. The present surface slopes toward the rift and King's Bowl.

To view part of the King's Bowl Rift underground, visitors walk on a trail blasted out of solid rock (Fig. 15-10). Exposed on the walls of the rift are vertical layers, called selvages. These were coated on the walls of the rift by chilling of the molten lava. Each layer represents one eruption followed by a draining of lava. In the vicinity of the King's Bowl, the rift is about six feet wide.

Crystal Ice Cave is a commercialized segment of the rift. It has a continuous ice floor and is 370 feet long. The clearness of many of the ice speleothems is breathtaking (Fig. 15-11). In some areas, ice crystals grow on the cave walls at certain levels (Fig. 15-12). Another cave south of Crystal Ice Cave and north of the King's Bowl contains a room 500 feet long, 40 feet wide, and 70 feet high. I know of no other cavern chamber approaching such dimensions. Fissure caves of this sort are uncommon, and I would appreciate information on any others known or discovered throughout the world.

MATHEMATICAL ANALYSIS OF SOME LAVA TUBE MECHANICS

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PART I — BETA FORMULA

The simplest form of tube-bearing flow occurs when a mass of lava spreads from a small source, coolings as it advances. Consider a differential element of length dy, width dx, and height z. Heat radiation from the top surface is $\frac{dU}{dT} = E\sigma T^4 dx\,dy$. Change in heat content is $dU = pcz\,dx\,dy\,dT$. Combining: $E\sigma T^4 dx\,dy\,dt = pcz\,dx\,dy\,dT$. Or, taking $v = \frac{dy}{dt}$, which defines the coordinate system: $dx\,dy = zv\,dx\,\frac{pc}{E\sigma}\frac{dT}{T^4}$, and integrating: $\iint dx\,dy = \int zv\,dx \int \frac{pc}{E\sigma}\frac{dT}{T^4}$, where the integrals on the right separate because T becomes independent of x in this coordinate system.

The double integral is surface area of the lava flow, while the first integral on the right is flow rate, Q. Defining beta as the temperature integral: 1) $A = \beta Q$, which is the desired result.

For a pahoehoe flow with constant, but not necessarily known, temperature profile, and neglecting heat of fusion: $\beta = \frac{pc}{E\sigma}\left(\frac{1}{T_1^3} - \frac{1}{T_0^3}\right)$. Using lack-of-movement freezing point for T_1 and eruption temperature for T_0, this describes a flow unit. Inserting Wentworth's temperature figures, beta is 5×10^3 sec/meter, which should be accurate to better than 30%.

Aa basalts have much lower surface temperatures than pahoehoes, so they should have significantly higher betas. Acidic lavas, on the other hand, have large T_1 and should have correspondingly small beta.

If flow rate is constant, equation (1) can be substituted into the volume equality $Ah = V = QT$, giving: (2) $\beta h = T$, which relates mean thickness of the flow unit to time of formation. For this relation it is convenient to use the reciprocal of beta, 70cm/hour.

It should be noted that a large number of fluid properties would intuitively be expected to affect size of a flow unit, but these properties all vanish with proper selection of coordinates and regions of integration. These properties affect shape, but not <u>size</u>.

It also should be noted that the T are surface temperatures, and that E can be eliminated by using pyrometer temperatures. This means that experimental error should be essentially the error in measuring Q. Equation (2) actually may be accurate enough that experimental beta could be used to obtain heat of fusion of the lava.

Formula (1) represents the conservation of energy, as applied to heat. The integral that gives beta is much less general. We might want to add terms for convection cooling, for heat lost from the bottom of the lava flow, or for heat of fusion, and we might want to assume temperature profiles that vary with surface temperature, depending on type of lava flow. If we are trying to predict size of flow units, fancy corrections don't mean much. We have a number for beta, and the corrections merely specify how the number was obtained. If we are observing sizes, then calculating properties, the corrections are what we are trying to calculate so fancier forms are necessary. My studies are not in that area.

PART II — GROWTH EQUATIONS OF THE SEMI-TRENCH RIDGE
(Introductory Note on the Formation of Lava Tubes, by Russell G. Harter)

A common conception of lava tube formation is generally stated approximately as follows: the upper surface of a lava flow hardens, and the still-liquid lava below drains out, leaving a lava tube. The tubes are cylindrical, and make a network of capillaries that can permeate the entire flow.

Many lava caves in the northwestern United States are quite complex, but this is readily accounted for by the common notion. It permits one to explain all arrangements of passages. Indeed, this concept enables one to explain any lava tube structure -- whether it is physically possible or not. My brother and I have abandoned this concept.

When lava flows across a surface, it tends to move in well defined channels. There are several types of lava channels, and lava tubes are roofed lava channels. Each is built in a definite way. Lava tubes do not happen suddenly nor miraculously, and are not impossibly complex.

When lava erupts, it flows out of a large crack in the ground. The crack (rift) often trends downhill from the point of the eruption, so the lava may follow it. A current develops in the rift, making it a lava channel. The top surface of the lava chills, making a thick, mushy scum. The scum is separated from the molten lava by the accumulation of a layer of hot gas rising from the lava. The hot gas supports the crust until it cools enough to stand by itself. Now the lava may drain away, leaving a cave. Since this produces a cave in a rift, it is a rift cave. This type of lava tube cave is almost always high and narrow--the shape of the rift. False roofs may form at low stages of flow, separating the rift into levels that are perfectly superimposed, one above another. Rift caves are hollow dikes.

If a lava flow is not in a rift, the lava streams must form their own walls. Thin sheets of lava flow out to the sides and cool, building walls along the channel. Or, in areas of low ground slope where the lava is temporarily ponded, the stream channel continues through it. This latter case we call a true trench, and the case of the lava stream forming a broad ridge through building its own levees we call a semitrench. True trenches leave a flat surface because they are incised into a flow unit by differential solidification.

Most large lava tubes are rift caves or semitrenches. They are able to transport hot lava for long distances, because comparatively little heat is lost from the roofed channels.

Large lava tubes feed small streams of hot lava. A single arched crust forms over the stream, making a smaller lava tube. This kind of tube is entirely above the adjacent ground surface, so we call it a surface tube. It is described by the beta formula just discussed. Most small lava tubes are surface tubes or a hybrid of surface tube and semitrench.

Semitrenches often flow in braided lava streams, much like the braided channels of a water stream. The individual channels over-flow requently, sometimes making surface tubes. In the resulting cave, the surface tube is a small lead off of a large passage. Overflows through the roof make surface tubes on an upper level. If the overflow plugs, it will leave a cupola in the ceiling of the lower passage.

After lava tubes form initially, rock is often added or removed, changing the passage shape. The lava is added as layers of linings. Rock is removed primarily by rockfalls. The many passage combinations, and extreme modifications, result in caves that are sometimes very complicated. They are possible to explain, however, without resort to illusion.

Lambda Relations of Semi-trench Growth

A lava flow advances slowly, pushing out small lobes and thin sheets of lava. Farther uphill, we find that the great bulk of the lava flow is solid. There is molten lava only at the tip and in a narrow longitudinal channel.

Later, after the lava flow has cooled, we find that the uphill ends of the sheets and lobes lie less than five centimeters behind the surface of the channel wall. This is quite remarkable: the structure of the growth region appears random, but both channel and lava flow have uniform shape and size, for extensive distances.

Clearly, growth of the lava flow must be a process that is uniform in the large but not in the small, and it must be inherently stable. The stability requirement can be written as a differential equation:

$$1)\quad \frac{\partial^2 y}{\partial x \partial T} \cdot \frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial z \partial T} \tan\theta = 0,$$

where y is thickness, x is width, z is length, and θ is ground slope. This equation states that deposition rate is a function of position along the flowline.

The uniformity requirement is a change of variable: $\mathfrak{z} = (vT - z)\cot\theta$, where v is rate of advance of the lava. 1) becomes: $\frac{\partial^2 y}{\partial x \partial \mathfrak{z}} \cdot \frac{\partial y}{\partial x} = \frac{\partial^2 y_x}{\partial \mathfrak{z}^2}$

This equation will integrate directly, once: $\left(\frac{\partial y}{\partial x}\right)^2 = 2\frac{\partial y}{\partial \mathfrak{z}} + F(x)$ The arbitrary function F must be evaluated by use of boundary conditions. This is a somewhat tedious process, because the bounds concern lack of roots of derivatives, but the result is $F(x)$ is a constant. This implies that y has the form $y = y^* + C$. We work with y^*, and define p: $\frac{\partial y^*}{\partial x} = p$, $\frac{\partial y^*}{\partial x} = \frac{1}{2}p^2$

Inverting variables: $\frac{\partial x}{\partial \mathfrak{z}} / = -p$, $x = 6' - p\mathfrak{z}$

$$y^* = \int p \frac{\partial x}{\partial p}\, dp, \quad y^* = p6' - 6 - \tfrac{1}{2}p^2\mathfrak{z}$$

where 6 is any function in p of class C_2, and the primes are derivatives with respect to p.

Again, boundary conditions must be appled. This time, we find that a suitable choice of origin gives $6 \equiv 0$: $x = -p\mathfrak{z}$, $y = C\mathfrak{z} - \frac{p^2\mathfrak{z}}{2}$

$$y = C\mathfrak{z} - \frac{x^2}{2\mathfrak{z}}$$

Since we are dealing with a ridge, and not a trench, C must be positive. Let $C = \frac{2}{\lambda^2}$. Also define $x_0 = \lambda y_0$ and $\mathfrak{z}_0 = \frac{\lambda}{2}x_0$. Then:

$$2)\quad y = y_0\frac{\mathfrak{z}}{\mathfrak{z}_0} - \frac{x^2}{\lambda^2 y_0}\frac{\mathfrak{z}_0}{\mathfrak{z}}$$

This function is a parabolic cone with vertex at the origin. It has two features that we can be quite sure that the actual lava flow does not have : it is smooth, and the leading end has a definite corner. Both of these were introduced in the boundary conditions, as a requirement that we are dealing with a mean surface and ignoring individual flow lobes. Less restrictive F would give full detail of the flowfront, but the second integration would have to be performed numerically, on a computer. Exact solution would be impossible.

Lambda now must be evaluated. We begin by specifying that the point $(x_0, 0, z_0)$ is the end of the growth region. For this z:

$$3)\quad y = y_0 - \frac{x^2}{\lambda^2 y_0}$$

and the flowline through the point is $x = x_0 e^{-\frac{\mathfrak{z}}{\mathfrak{z}_0}}$, which gives the growth region an area: $A_G = 3x_0\mathfrak{z}_0\tan\theta = \frac{3}{2}\lambda^3 y_0^2\tan\theta.$

For a channel with liquid depth h, area A_1, and shape factor N, flow is: $Q = \frac{\rho g}{3\mu}A_1 h^2 N\tan\theta$ (For constant width w, $N = 1/(1 + (2h/w)^2)$.) Substituting A_G and Q into the beta formula gives:

$$4)\quad \lambda^3 = \frac{A_1}{A_0}N\left(\frac{h}{y_0}\right)^2$$

where A_0 is a collected constant with dimensions of area: $A_0 = \frac{9\mu}{2\rho g}\beta$.

Since A_0 depends on viscosity of the lava, care must be taken in using any fixed value. However, 7.2 mm^2 should be accurate enough for finding lambda from dimensions of a lava tube. In other work, such as predicting how width of a lava flow is going to vary at a change of slope, one of the A_G forms might be preferable.

To this point, all calculations have concerned the initial-growth portion of the lava flow. However, the form of 2) shows that equations 3) and 4) will still apply at later stages of development. The term h/y_0 is the key factor. For an open channel, it equals 1. For a lava tube, $y_0 - h$ is the roof thickness, and the term may make an appreciable correction.

V has been ignored. Since it is rate of advance of the lava flow, someone may want to calculate it: $v = \frac{3\lambda^2}{2\beta}\tan\theta$.

This development is not quite complete. One additional equation is needed, before application to active lava flows will be feasable. This equation would relate one of the other parameters to theta. It probably can be obtained from energy considerations, but the proper energy function definitely is not obvious.

Without the additional equation, 3) and 4) can be used to find obscured flow edges, to determine whether all major tubes of a lava flow have been found, and to locate any missing ones. For this work, A_o=7.2mm^2 probably is not accurate enough, and an experimental value probably should be found by measuring actual lava-tube flows.

I found equation 1) about the beginning of July, so I haven't had time to check the value of A. This would make a good project for someone who is feeling frustrated because our magnetic caves are making his Brunton point east. The surveying is all tape and verticals, and you can omit the verticals if you pick something like the Red Cave system to measure. Chain off the width of the lava flow to get X_o and the depth of the collapse to get Y_o0--remember that you have to go to the bottom of the breakdown. Then lambda is X_o over Y_o. Measure passage size and shape, and that gives h, A_1 and N. We have a graph (Fig.). These curves are made for shape of a semitrench passage, rather than for a rectangular one, which is what the essence of N_s is all about.

The most immediate application of this development is tracing lava tubes in stretches where cave is not known. We know that a semitrench lava tube is going to lie on the axis of a ridge. Equation 3 describes the shape of the ridge, and equation 4 gives its width. This is sufficient to allow one to begin at a cave and proceed along the ridge until he reaches the next cave perhaps a half-mile distant.

We do not have a good way to state what parts of a system will be found to be collapsed, what parts are going to form caves, and what parts are going to be plugged with solid lava. But a little practice allows us to follow plugged or obscured segments almost as easily and accurately as we follow collapsed segments.

Ridge-walking is, of course, an old technique. What the mathematics do for us is to relate the ridge and the cave. Included is warning that the ridge shape may lead one astray, and provision to get back on line when we do stray.

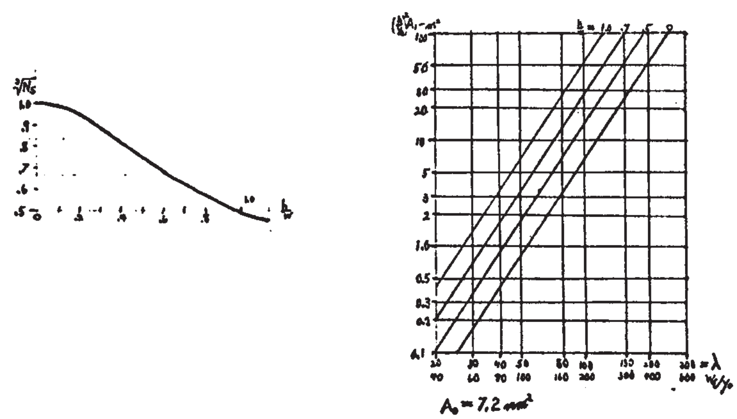

Figure 16-1: Graph for Lambda relations of Semi-Trench Growth.